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Thermodynamic AI and Thermodynamic Linear Algebra

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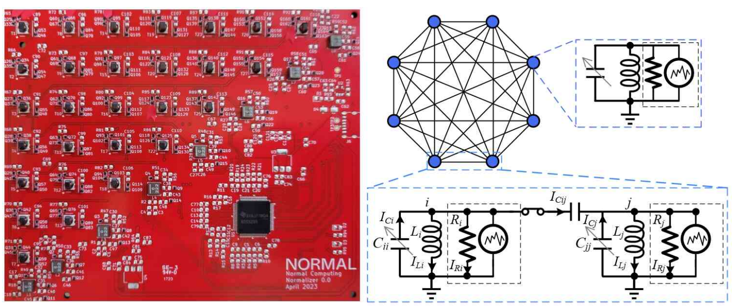
NEURAL INFORMATION PROCESSING SYSTEMS

Many Artificial Intelligence (AI) algorithms are inspired by physics and employ stochastic fluctuations, such as generative diffusion models, Bayesian neural networks, and Monte Carlo inference. These algorithms are currently run on digital hardware, ultimately limiting their scalability and overall potential. Here, we propose a novel computing device, called Thermodynamic AI hardware, that could accelerate such algorithms. Thermodynamic AI hardware can be viewed as a novel form of computing, since it uses novel fundamental building blocks, called stochastic units (s-units), which naturally evolve over time via stochastic trajectories. In addition to these s-units, Thermodynamic AI hardware employs a Maxwell's demon device that guides the system to produce non-trivial states. We provide a few simple physical architectures for building these devices, such as RC electrical circuits. Moreover, we show that this same hardware can be used to accelerate various linear algebra primitives. We present simple thermodynamic algorithms for (1) solving linear systems of equations, (2) computing matrix inverses, (3) computing matrix determinants, and (4) solving Lyapunov equations.

Stochastic Processing Unit

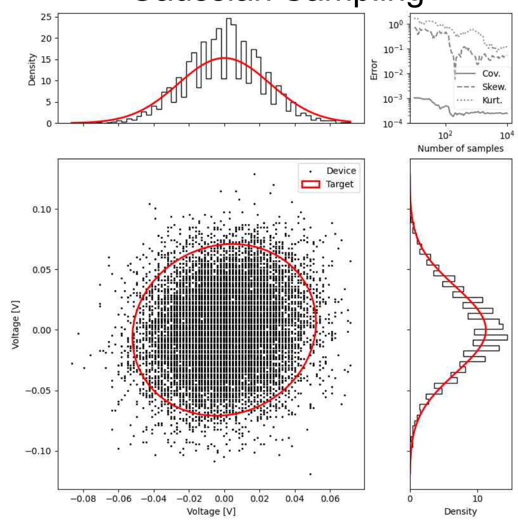


(Left panel) The Printed Circuit Board for our 8-cell SPU. (Right panel) Illustration of eight unit cells that are all-to-all coupled to each other, as in our SPU. Each cell contains an LC resonator and a Gaussian current noise source, as shown in the circuit diagram on the top right. The circuit diagram on the bottom depicts two capacitively coupled unit cells.

$$\mathrm{d}I = \mathbf{L}^{-1} V \mathrm{d}t$$
$$\mathrm{d}V = -\mathbf{C}^{-1}\mathbf{R}^{-1} V \mathrm{d}t - \mathbf{C}^{-1} I \mathrm{d}t + \sqrt{2\kappa_0}\mathbf{C}^{-1}\mathcal{N}[0, \mathbb{I}\,\mathrm{d}t],$$

The SPU can be mathematically modeled as a set of capacitively-coupled ideal LC circuits with noisy current driving.

Gaussian Sampling



The Hamiltonian for the coupled oscillator system is given by:

$$\mathcal{H}\left(\vec{I}, \vec{V}\right) = \frac{1}{2}\vec{V}^T \mathbf{C} \vec{V} + \frac{1}{2}\vec{I}^T \mathbf{L} \vec{I},$$

where $\vec{I}$ is the vector of currents through the inductors in each unit cell, $\vec{V}$ is the vector of voltages across the capacitors in each unit cell, C is the Maxwell capacitance matrix and L is the inductance matrix

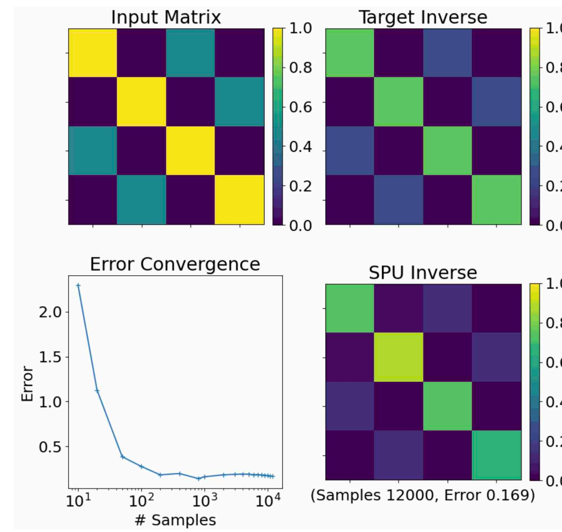
At thermal equilibrium, the dynamical variables are distributed according to a Boltzmann distribution, and hence $\vec{V}$ is normally distributed:

$$\mathcal{N}(\vec{x}|\mathbf{\Sigma}) = \frac{1}{\sqrt{(2\pi)^N |\mathbf{\Sigma}|}} \exp\left(-\frac{1}{2}\vec{x}^T \mathbf{\Sigma}^{-1} \vec{x}\right)$$

The measured covariance matrix is related to the input Maxwell capacitance matrix.

$$C = k_B T \ \Sigma^{-1}$$

Thermodynamic Matrix Inversion



The input matrix A and its true inverse A^{-1} are shown, respectively, on the top left and top right. The relative Frobenius error versus the number of samples is plotted in the bottom left. The bottom right shows the experimentally determined inverse after gathering 12000 samples from the SPU.

References

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